MATHEMATICS TEACHER
Learning & Teaching
PK 12

Mathematics Teacher: Learning and Teaching PK-12, is NCTM's newest journal that reflects the current practices of mathematics education, as well as maintains a knowledge base of practice and policy in looking at the future of the field. Content is aimed at preschool to 12th grade with peer-reviewed and invited articles. *MTLT* is published monthly.

ARTICLE TITLE:

Building Embodied Intuition for Graphs With Smartphones

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DIGITAL OBJECT IDENTIFIER: 10.5951/MTLT.2024.0297

VOLUME: 119

ISSUE NUMBER: 1

Mission Statement

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Approved by the NCTM Board of Directors on July 15, 2017.

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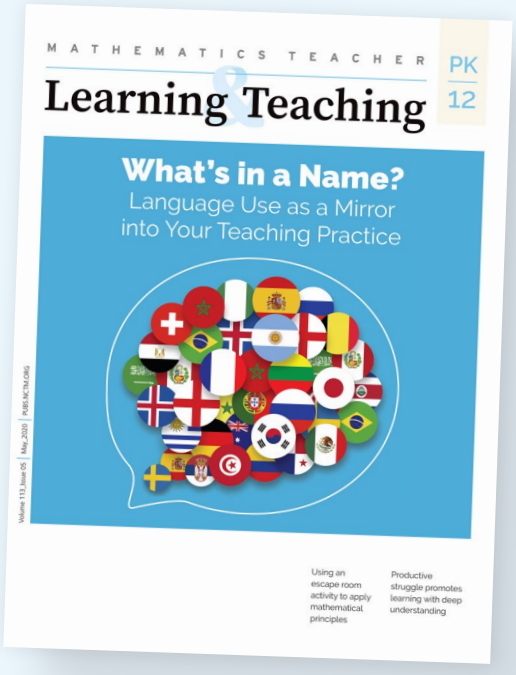

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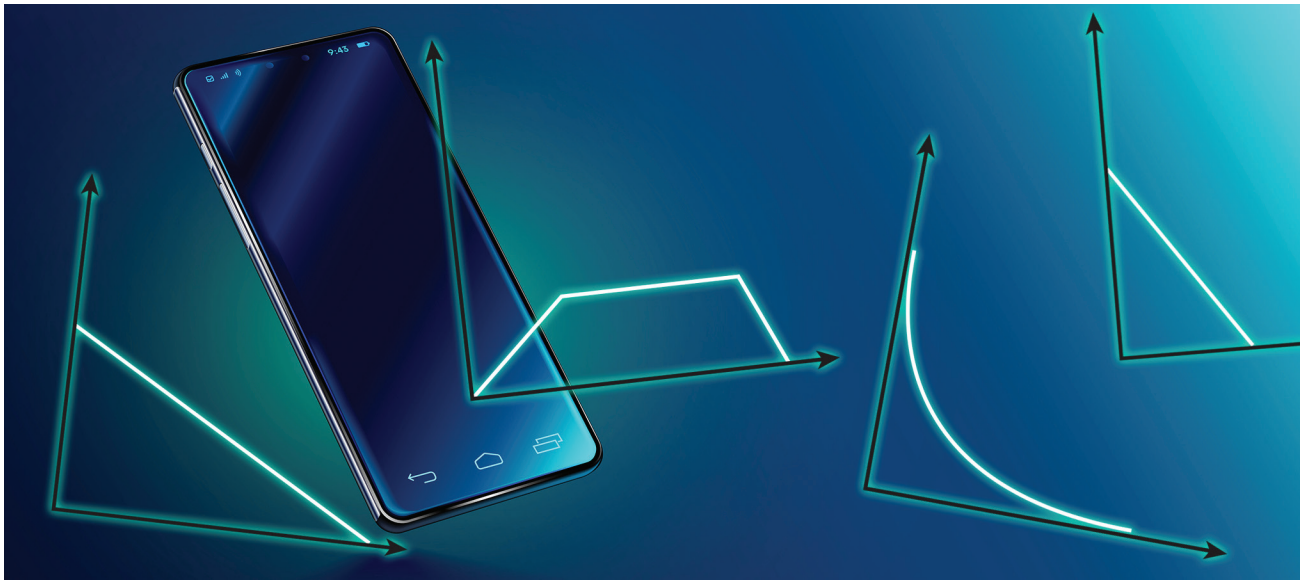


Building Embodied Intuition for Graphs With Smartphones

LiDAR technology helps students relate body-based motion to its mathematical representations.

Rebecca E. Vieyra, Colleen Megowan-Romanowicz, Daniel O'Brien, Chrystian Vieyra Cortés, and Mina C. Johnson-Glenberg

Mathematics is a powerful tool for modeling motion: the change in speed of a falling rock, or the curve of a kicked ball. While motion is a popular context for applying linear and quadratic functions, research demonstrates that students struggle to connect mathematical representation of motion—especially graphical representations—to the real world (Börner et al., 2019). Difficulties understanding motion are typically perceived as the domain of science education (e.g., McDermott et al., 1987), but frequent use of motion graphs in mathematics suggests that this should be of concern to mathematics educators and researchers as well.

Some students' difficulties with motion pertain to language use. For example, students might not articulate the difference between position (location on a number line) and distance (cumulative units traveled); time ("clock-reading") and time interval

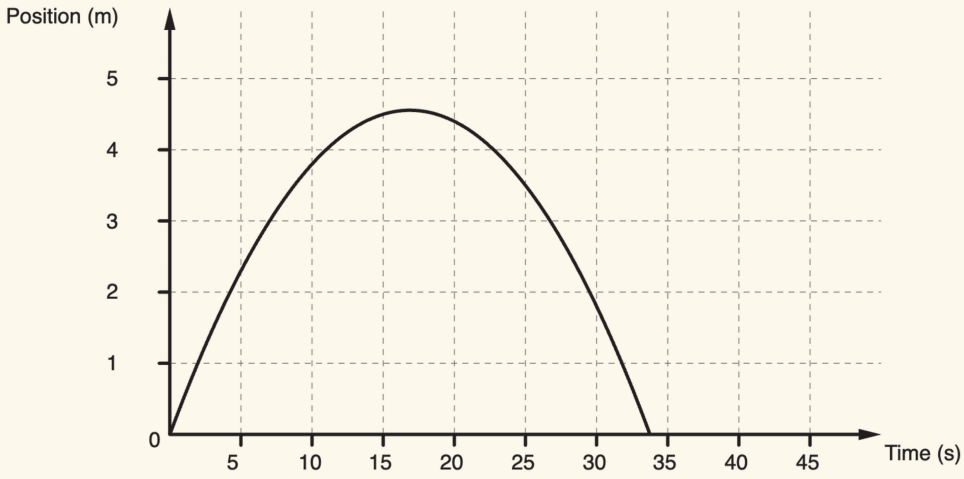
(elapsed time); or speed (scalar of how fast an object moves), velocity (vector representing speed and direction), and acceleration (change in speed and direction). Teachers' frequent use of the generic phrase "rate of change" may compound this issue (Teuscher & Reys, 2010). If teachers do not press students to use explicit language to describe motion, they may not uncover persistent misconceptions.

Many students also misinterpret time-based graphs of linear motion as two-dimensional pathways. Students typically describe the motion in Figure 1 as "an object climbs up a hill, then comes down the other side," rather than the correct interpretation that "an object moves quickly away from the origin and then slows down, pauses instantaneously, and then speeds up as it returns to the origin." Some teachers and even textbook authors reinforce students' naive beliefs by using imprecise language (Mazibe et al., 2020).

EASING LEARNING CHALLENGES WITH SMARTPHONES

An approach we developed to address these challenges is to have students "walk the graph" while getting real-time feedback using smartphones. Walking the graph entails students moving their bodies toward or away

Figure 1 Position vs. Time Graph for an Object



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doi:10.5951/MTLT.2024.0297

from a wall (Figure 2), matching their position, speed, and direction with a position-time graph. This activity provides the teacher with immediate feedback on how a student interprets a graph, so that misconceptions can be addressed promptly. It also leverages embodied learning. Research in embodied learning has shown that cognition is not confined to the brain—it is integrated with the sensorimotor system and grounded in action and perception (Lindgren & Johnson-Glenberg, 2013).

Researchers have explored the role of embodied cognition—the idea that bodily experiences influence sense making—in various domains, including mathematics (Alibali & Nathan, 2012) and physics (Johnson-Glenberg & Megowan-Romanowicz, 2017), particularly when spatial reasoning is necessary. They recommend having students act out the motion depicted in a graph with their bodies, relating different graphs of the same kind of motion (such as position-time, velocity-time, and acceleration-time graphs), and using these representations to predict motion. Students who move their bodies as they study motion can leverage physical inputs, such as the exertion of their muscles or the air moving past their skin, to make sense of what they are doing (Lindgren & Johnson-Glenberg, 2013). What students say as they engage in body-based mathematics activities provides opportunities for student collaboration (Marshall & Carrejo, 2008).

The sensors in many smartphones can maximize students' ability to graph motion using their body movements, helping them develop intuition for motion graphs. While bringing motion sensors into the mathematics classroom is not new (Urban-Woldron, 2012), doing so is now more accessible given smartphones' prevalence. Our team created an app that uses the LiDAR sensor in modern iPhone Pro and iPad Pro models to help students understand motion graphs. The app, Physics Toolbox Sensor Suite, is a free multi-sensor education tool available from the Apple App Store (Vieyra, n.d.), and the LiDAR mode is accessible from the side menu: Kinematics >> Motion Visualizer. The app uses back-facing infrared beams to measure the distance to an object, such as a flat wall, and plots graphs of position-time and velocity-time as the user moves toward and away from the wall (Figure 3). While commercial education sensors, such as those from Texas Instruments, PocketLab, Vernier, and PASCO, have sonic or infrared motion rangers, they are uncommon in mathematics classrooms, typically have limited ranges, and are not necessarily optimized to enhance students' graph-matching abilities.

Along with the ability to produce graphs of a student's motion, the app includes a 14-level game that challenges users to walk out different kinds of motion (Figure 4). Students first match graphs by walking toward the wall, first at a slow constant speed (Level 1) and then again at a faster constant speed (Level 2), noting differences in the graph's slope. Subsequently, students graph their motion when walking away from the wall (Level 5) and as part of multi-segmented motion (Level 7). Across 20+ classrooms, we have seen that learners are particularly challenged when trying to match curved position-time graphs, representing acceleration. However, as they apply what they learn

Figure 2 Wall and Number Line

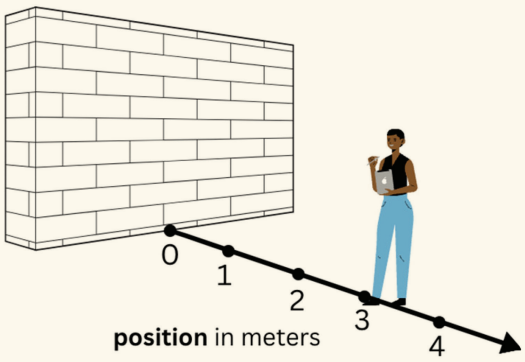
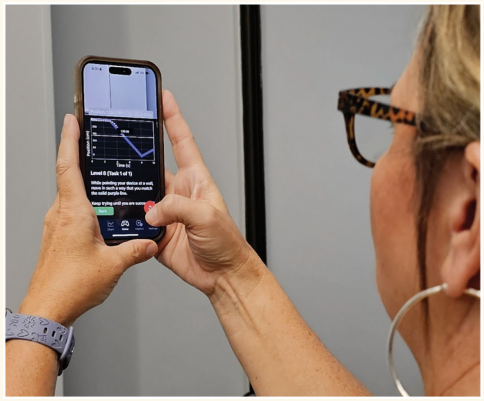


Figure 3 Math Teacher Walks a Graph



about interpreting the shapes of graphs in earlier challenges, almost all students can develop strategies to match the most difficult graphs. When students adequately match the graph on each level, they are rewarded with digital confetti. (See a video from NCTM TV showcasing the use of the app during NCTM 2024 [link online].)

TESTING THE APP IN CLASSROOMS

While the app was originally developed for high schools, our experiences show that students as young as fifth grade can benefit from thinking about motion graphs, with peak engagement in middle grades. Motion graphs can help students build fundamental ideas within the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and apply them to real-world motion, including ratios and proportional relationships (6.RP.3, 7.RP.2d), number systems (6.NS.5, 6.NS.6), dependent relationships and functions (6.EE.9, 8.EE.5, 8.F.2, 8.F.5), and foundations for modeling practices. These app-based motion challenges lead to greater conceptual growth than the use of commercial education sensors (Megowan-Romanowicz et al., 2023), and students—including some with learning and physical differences—are more engaged than when they learn via traditional instruction (Vieyra et al., 2024).

In 2024, we investigated whether middle school students could make sense of motion graphs with the app. We worked with four seventh-grade classes ($N = 98$) at a racially diverse Midwestern school. Students took a 12-question pretest to measure their existing understanding of motion graphs, a subset of questions from research-validated diagnostic tests (e.g., Beichner, 1994). Across two days, pairs of students completed the 14 challenges, alternating between being the "holder" who held the smartphone and walked out the graph and being the "helper" who provided guidance to the holder on how to walk and spot-checked for safety to ensure that the holder had clear space to walk. Pair-based activities were punctuated with whiteboard discussions led by two authors (Vieyra and Megowan-Romanowicz), which took place midway through the challenges, and then again at the end. Following these activities, students completed a posttest to measure change in conceptual understanding. The protocol used with these students is available on our site (link online).

SENSE MAKING THROUGH PAIR-BASED DIALOGUE AND COLLABORATION

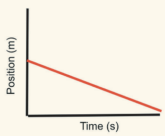
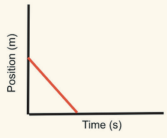
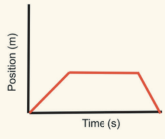
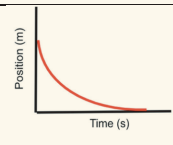
The following is a snapshot of dialogue between two students, Hana and Sofia, showcasing their learning about the app and the motion they are expected to act out. After familiarizing themselves with the app, they began working on the challenges. Sofia held the phone, and they watched the screen intently as the timer counted down to begin to match the first graph (refer to Level 1 in Figure 4).

Hana: Ready, set, go! [*Sofia moves tentatively toward the wall, but her graph doesn't quite match.*] Faster . . . no . . . go slower . . . no . . . sorry . . . oh, let's try again . . .

Sofia: I wasn't ready when it started. Wait, where do I need to be standing? . . . I need to get in position for the next time it starts. [*They get set for the next trial. Hana's hand rests lightly on Sofia's back to guide her movements.*]

Hana: Okay . . . three, two, one, move! [*Sofia gets off to a quick start.*] . . . try a little slower . . . better . . .

Figure 4 Selection of Graphs That Students Must Match

	Level 1	Level 2	Level 7	Level 12
Graph to match				
Required movement	Move toward the wall at a steady, slow pace.	Move toward the wall at a steady, fast pace.	Move away from the wall slowly, stop, move toward the wall quickly.	Starting at a fast pace, move toward the wall while slowing down.

hmmm. [*Sofia's graph flattens out as she approaches the wall.*] Try again.

Sofia: My line was too steep, and when I slowed down, my line got flatter. Did you see that? [Figure 5a]

Hana: Quick, back to the starting position! Three, two, one, now! Good! Almost . . . yay! We did it! [*They jump up and down excitedly as they see the burst of digital confetti.*]

Sofia: [*She smiles and hands over the phone.*] Now you . . .

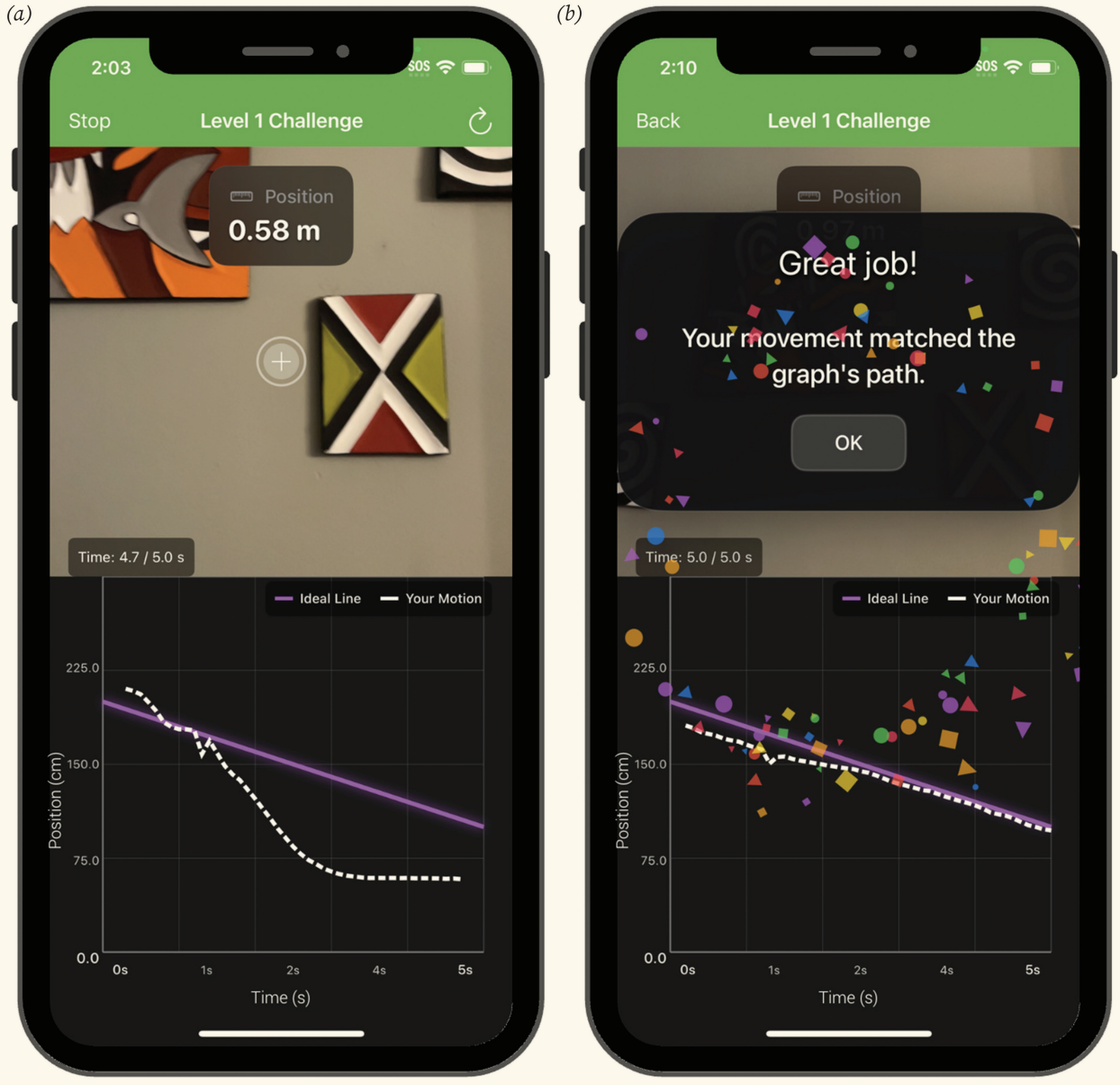
Hana: Okay. Where did you stand? [*Sofia points at the position meter above the graph and pulls Hana back a step.*]

Sofia: You have to hold the phone straight up and down so it measures the correct distance from the wall. [*Sofia corrects Hana's grip on the phone so it is vertical.*]

Hana: Three, two, one. [*Hana starts moving toward the wall at a constant speed.*] Oh . . . almost . . .

Sofia: Did you see how your slope was steeper than the graph you are trying to match? You were moving too fast. Get ready to go again.

Figure 5 *(a)* Sofia's Unsuccessful Attempt, *(b)* Hana's Successful Attempt



Hana: [*Hana steps back to her starting position and waits for the next cycle to begin.*] Three, two, one. [*Hana begins moving toward the wall. Her graph looks good. Confetti appears.*] Yes! [Figure 5b]
Sofia: Yay!

In Hana and Sofia's early attempts, they focused on getting the initial position correct and controlling the speed at which they moved. By the end of the first six challenges, they had mastered these strategies and developed an understanding of the relationship between the graph's slope and the speed and direction they must walk. When they reached challenge seven (refer to Level 7 in Figure 4), they were confronted with something new: a graph with three segments.

Hana: Oh . . . wait . . . it's flat in the middle . . . [*They both stare at the screen for a moment, and then Sofia points to the second segment.*]
Sofia: There. You stop there. The position stays the same for a couple seconds. Then you go back. [*Sofia takes the phone and gets ready to begin.*]
Hana: Three, two, one . . . [*Sofia begins backing slowly away from the wall and then gradually increases her speed as she sees that the slope of her graph is too shallow.*] Stop! [*Sofia stops.*] Go! [*Sofia moves toward the wall, gradually increasing her speed until the trial ends.*]
Sofia: Well, that was pretty bad. [*They giggle.*] So, I need to go faster at first, and then I need to be ready to move right when the flat spot ends . . .
Hana: Yep. [*Sofia walks back to her starting position.*] Three, two, one . . . [*Sofia moves backward with a little more assurance this time, then stops.*] Now! [*Sofia starts moving again back toward the wall, but realizes she is moving too quickly and slows down.*] Well, at least the first parts were close. Go again.
Sofia: The last part is not as steep as the first part. I need to move a little slower. [*She walks back to the starting position and waits for the timer. When the graph starts, they both move, first backward, then they stop, then forward. Confetti!*] Yes! [*She pumps her fist and hands the phone to Hana, both of them smiling broadly.*]

EVIDENCE OF IMPACT

As shown in Figure 6, we aimed to consolidate learning along the way with whiteboarding discussions (Megowan-Romanowicz, 2016), an approach frequently used by mathematics and science teachers who use Modeling Instruction (MI) pedagogy. MI is a research-based, student-centered method of teaching originally developed for use in high school physics classrooms, where kinematics is the first fundamental conceptual model students must acquire (American Modeling Teachers Association, n.d.; Wells et al., 1995).

Whiteboarding discussions help students build a consensus view of the phenomenon under study and foster the notion in students that *knowledge resides in their peers,* not just in their teachers. In MI, students learn to engage in Socratic dialogue, asking

Figure 6 Students Present Whiteboards to the Class

Credit: Aaron Mueller.

each other questions to seek clarification, make corrections, and build a common conceptual model. Careful facilitation ensures group collaboration during whiteboard preparation and individual accountability during whiteboard-mediated discussions, supporting engagement by all and a sense of personal agency among students. This shift of classroom norms encourages the students to do the talking (Megowan-Romanowicz et al., 2017).

Following their work with the app, students were asked to share what they learned by constructing a multi-segment graph on their whiteboard and making up a story that described the motion it displayed. Working in groups of four, students created a position-time graph illustrating at least five sections of unique motion, with at least one nonlinear curve. Students self-corrected as they discussed interpretations of their graphs. For example, in Figure 7, students labeled the first segment as "constant speed." When asked what was meant by this, students engaged in a lively discussion, using evidence from their engagement with the app:

Sid: That part of the graph shows that the person would walk away from the wall at a steady pace. [*Sid draws the marker along the graph, showing how position changes as time elapses.*]

Manny: That's not right. The flat line means that time went by, but the position didn't change.

Anna: The position didn't change. But that's still a constant speed. It's just that it's a constant speed of zero!

Multiple groups added additional story content. Figure 8 illustrates one group's story of a character named Mary who left her home, stopped to talk to her mom in the front yard, and continued walking away from home until she noticed she forgot her phone, and then returned to the house. Students demonstrated their story by walking out the motions. In this particular scenario, students noticed something unusual about the third and fifth motion segments.

Maya: You said in segment 3 that "she walked away from her mom, picking up speed as she walked." I agree that she picked up speed, at first—the graph curved up from segment 2 to segment 3. But what about when she heads toward segment 4?

Teacher: Interesting. Let's look at segment 4. What's happening right at the maximum of the graph?

Nancy: She's turning around.

Teacher: Okay, now tell me about her speed.

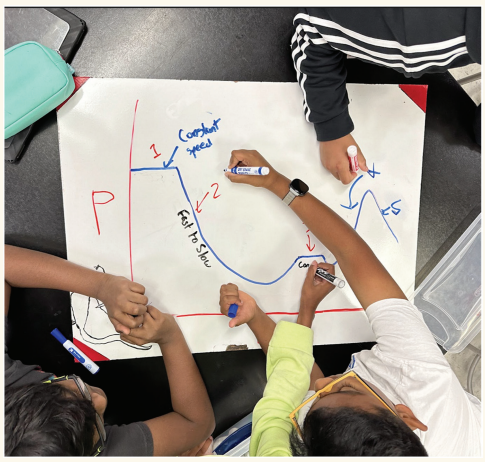
Maya: Halfway through segment 3, the graph goes toward horizontal.

Teacher: The slope is going toward zero. [*Teacher holds out arm horizontally.*] What does it mean a person is doing when their position-time graph is flat?

Nancy: It means she stopped, but just for an instant.

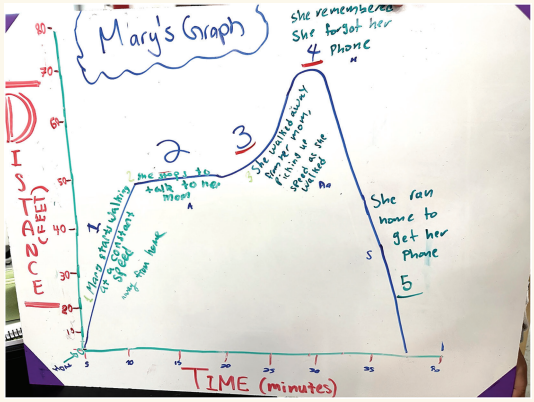
Maya: So that means that during segment 3, she must have sped up, and then she started to slow down.

Figure 7 Students Create a Position-Time Graph in a Small Group



Credit: Aaron Mueller.

Figure 8 A Group's Whiteboard With a Detailed Storyline



This whiteboard also included a comparison of segments 1 and 5. Segment 1 is labeled as Mary "walking at a constant speed," while segment 5 is labeled as "running home." Both segments display equal speeds because the magnitude of their slopes was the same, although one was positive and the other negative. During the whiteboarding session, they were prompted to clarify how they could distinguish between different speeds in different directions, and, as needed, to make corresponding modifications to either the graph or the verbal description so that ultimately both representations "told the same story" about how Mary moved.

Whiteboard discussions, based on students' experiences with the app, helped them use explicit language to communicate precise descriptions of rates of change (in distance, position, speed, velocity, and acceleration). They articulated physical interpretations of slopes, graphical intercepts, curves, and graphical features such as maxima and minima. Students also realized that they could not rely on language that expressed motion in terms of an absolute value, such as "speed," but needed to specify the direction of motion to convey their understanding fully, thus setting the stage for grasping the distinction between scalar and vector quantities. Having seen the velocity-time graphs in the app, multiple groups also expressed curiosity about how to re-graph their position-time graphs as velocity-time graphs, laying the foundation for a discussion of functions and their derivatives. Students demonstrated that they understood a point on the graph as an object's position at an instant in time, even going so far as to say that a vertical line was "not possible" because it would mean that an object could be in more than one location at the same time.

For a first formal exposure to motion graphs, students demonstrated substantial growth. Quantitatively, students' performance on the conceptual posttest ($M = 8.13$, $SD = 2.39$) showed an increase from that on the pretest ($M = 5.96$, $SD = 2.32$), illustrating a significant improvement: $t(98) = 18.68$, $p = .001$. Qualitatively, students' ability to master the app challenges, and afterward create motion graphs with accurate verbal descriptions, demonstrated their ability to apply the embodied learning by "walking the graph."

GUIDELINES FOR USING THE APP IN CLASSROOMS

We encourage teachers to integrate the activity at moments in which students need to make meaning of functions and their graphs. Teachers are welcome to modify our protocol, and we encourage them to help students contrast graph shapes and their corresponding motion while building motion vocabulary. Teachers should also consider school or district policies regarding smartphone use.

Motion provides ample opportunity for introducing and reinforcing critical elements of mathematical modeling with graphs. However, teachers who use motion as a context to teach mathematical modeling should do so with care, providing students with embodied experiences and sufficient time to engage in sense making. We hope that our smartphone app and protocol provide fertile ground for students to make relevant, real-life connections to mathematics.

REFERENCES

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the Learning Sciences, 21*(2), 247–286. https://doi.org/10.1080/10508406.2011.611446

American Modeling Teachers Association. (n.d.). *What is AMTA?* https://www.modelinginstruction.org/

Beichner, R. (1994). Testing student interpretation of kinematics graphs. *American Journal of Physics, 62*(8), 750–762. https://doi.org/10.1119/1.17449

Börner, K., Bueckle, A., & Ginda, M. (2019). Data visualization literacy: Definitions, conceptual frameworks, exercises, and assessments. *Proceedings of the National Academy of Science, 116*(6), 1857–1864. https://doi.org/10.1073/pnas.1807180116

Johnson-Glenberg, M. C., & Megowan-Romanowicz, C. (2017). Embodied science and mixed reality: How gesture and motion capture affect physics education. *Cognitive Research: Principles and Implications, 2*(4), 1–28. https://doi.org/10.1186/s41235-017-0060-9

Lindgren, R., & Johnson-Glenberg, M. (2013). Emboldened by embodiment: Six precepts for research on embodied learning and mixed reality. *Educational Researcher, 42*(8), 445–452. https://doi.org/10.3102/0013189X13511661

Marshall, J. A., & Carrejo, D. J. (2008). Students' mathematical modeling of motion. *Journal of Research in Science Teaching, 45*(2), 153–173. https://doi.org/10.1002/tea.20210

Mazibe, E. N., Coetzee, C., & Gaigher, E. (2020). A comparison between reported and enacted pedagogical content knowledge (PCK) about graphs of motion. *Research in Science Education, 50*, 941–964. https://doi.org/10.1007/s11165-018-9718-7

McDermott, L. C., Rosenquist, M. L., & van Zee, E. H. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics. *American Journal of Physics, 55*, 503–513. https://doi.org/10.1119/1.15104

Megowan-Romanowicz, C. (2016). Whiteboarding: A tool for moving classroom discourse from answer-making to sense-making. *The Physics Teacher, 54*(2), 83–86. https://doi.org/10.1119/1.4940170

Megowan-Romanowicz, C., Dukerich, L., & Posthuma-Adams, E. (2017). Let the students do the talking. In K. D. Kloepper & G. L. Crawford (Eds.), *Liberal arts strategies for the chemistry classroom* (pp. 71–97). American Chemical Society.

Megowan-Romanowicz, C., O'Brien, D., Vieyra, R., Vieyra Cortés, C., & Johnson-Glenberg, M. (2023). Evaluating learning of motion graphs with a LiDAR-based smartphone application. *2023 PERC Proceedings*, 224–229. https://doi.org/10.1119/perc.2023.pr.Megowan-Romanowicz

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. https://www.thecorestandards.org/Math/

Teuscher, D., & Reys, R. E. (2010). Connecting research to teaching: Slope, rate of change, and steepness: Do students understand these concepts? *Mathematics Teacher, 103*(7), 519–524. https://doi.org/10.5951/MT.103.7.0519

Urban-Woldron, H. (2012). Motion sensors in mathematics teaching: Learning tools for understanding general math concepts? *International Journal of Mathematical Education in Science and Technology, 48*(4), 584–598. https://doi.org/10.1080/0020739X.2014.985270

Vieyra, C. (n.d.). *Physics toolbox sensor suite*. https://apps.apple.com/us/app/physics-toolbox-sensor-suite/id1128914250

Vieyra, R., Megowan-Romanowicz, C., Johnson-Glenberg, M. C., O'Brien, D., & Vieyra Cortés, C. (2024). Making motion meaningful: Mapping body movements onto graphs. *The Science Teacher, 91*(6), 57–64. https://doi.org/10.1080/00368555.2024.2404956

Wells, M., Hestenes, D., & Swackhamer, G. (1995). A modeling method for high school physics instruction. *American Journal of Physics, 63*(7), 606–619. https://doi.org/10.1119/1.17849

ACKNOWLEDGMENTS

This research was supported by National Science Foundation grant 2114586, a collaboration between the American Modeling Teachers Association and Arizona State University. It was carried out under Arizona State University IRBs #0018734 and #00018889. Student assent and parental consent were obtained for individuals involved in the study. Special thanks to Aaron Mueller and the students who participated in this experience.

